

QED correction to asymmetry for polarized ep -scattering from the method of the electron structure functions

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Abstract

The electron structure function method is applied to calculate model-independent radiative corrections to an asymmetry of electron-proton scattering. The representations for both spin-independent and spin-dependent parts of the cross-section are derived. Master formulae take into account the leading corrections in all orders and the main contribution of the second order next-to-leading ones and have accuracy at the level of one per mille. Numerical calculations illustrate our analytical results for both elastic and deep inelastic events.

1 Introduction

Precise polarization measurements in both inclusive [1, 2] and elastic [3, 4] scattering are crucial for understanding the structure and fundamental properties of a nucleon.

One important component of the precise data analysis is radiative effects, which always accompany the processes of electron scattering. The first calculation of radiative corrections (RC) to polarized deep inelastic scattering (DIS) was done by Kukhto and Shumeiko [5], who applied a covariant method of extraction of an infrared divergence [6, 7] to this process. The polarization states were described by 4-vectors, which were kept in their general forms during the calculation. It required tedious procedure of tensor integration over photonic phase space, and, as a result, led to very complicated structure of final formulae for RC. The next step was done in the paper [8], where additional covariant expansion of polarization 4-vectors over a certain basis allowed to simplify the calculation and final results. It resulted in producing the Fortran code POLRAD [9] and Monte Carlo generator RADGEN [10]. These tools are widely used in all current experiments in polarized DIS. Later the calculation was applied to the case of collider experiments on DIS [11, 12]. We applied this method also to elastic process in papers [13, 14].

However, the method of covariant extraction of infrared divergence is essentially restricted by the lowest order RC. All attempts to go beyond the lowest order lead to very large formulae, that

are difficult to cross check, or to a simple leading log approach [15]. The recent developments are reviewed in ref.[16].

The decision can be found in applying the formalism of the electron structure functions (ESF). Within this approach such processes as the electron–positron annihilation into hadrons and the deep inelastic electron–proton scattering in one photon exchange approximation can be considered as the Drell–Yan process [17] in annihilation or scattering channel, respectively. Therefore, the QED radiative corrections (RC) to the corresponding cross–sections can be written as a contraction of two electron structure functions and the hard part of the cross–section, see [18, 19]. Traditionally these RC include effects caused by loop corrections as well as soft and hard collinear radiation of photons and e^+e^- -pairs. But it was shown in Ref. [19] how one can improve this method by inclusion also effects due to radiation of one non-collinear photon. The corresponding procedure concludes in modification of the hard part of cross section that provides the exact accounting of the lowest order correction and leads to exit beyond the leading approximation. We applied this approach to the recoil proton polarization in elastic electron scattering in ref.[20]. In the present paper we calculate RC to polarized DIS and elastic scattering following ref.[20].

Section 2 gives a short introduction to the structure function method. There we present two known forms of the electron structure functions, namely, iterative and analytical, which resums singular infrared terms in all order into exponent. In this section we also obtain master formulae for observed cross sections. Leading log results are presented in Section 3. These results are valid both for DIS and elastic cases. We also use an iterative form of ESF to extract the lowest order correction, that can provide a cross-check through comparison with known results. In Sections 4 and 5 we describe the procedure of generalization the results for next-to-leading order in DIS and elastic cases. Numerical analysis is presented in Section 6. We consider kinematical conditions of current polarization experiments at fixed targets as well as collider kinematics. Some conclusions are made in the Section 7.

2 Electron Structure Functions

A straightforward calculation based on the quasireal electron method [21] can be used to write the invariant cross–section of the DIS process

$$e^-(k_1) + P(p_1) \rightarrow e^-(k_2) + X(p_x) \quad (1)$$

in the following form

$$\frac{d\sigma(k_1, k_2)}{dQ^2 d y} = \int_{z_{1m}}^1 dz_1 \int_{z_{2m}}^1 dz_2 D(z_1, L) \frac{1}{z_2^2} D(z_2, L) \frac{d^2\sigma_{hard}(\tilde{k}_1, \tilde{k}_2)}{d\tilde{Q}^2 d\tilde{y}} , \quad L = \ln \frac{Q^2}{m^2} , \quad (2)$$

where m is the electron mass and

$$Q^2 = -(k_1 - k_2)^2 , \quad y = \frac{2p_1(k_1 - k_2)}{V} , \quad V = 2p_1 k_1 .$$

The reduced variables which define the hard cross–section in the integrand are

$$\tilde{k}_1 = z_1 k_1 , \quad \tilde{k}_2 = \frac{k_2}{z_2} , \quad \tilde{Q}^2 = \frac{z_1}{z_2} Q^2 , \quad \tilde{y} = 1 - \frac{1 - y}{z_1 z_2} . \quad (3)$$

The electron structure function $D(z, L)$ includes contributions due to photon emission and pair production

$$D = D^\gamma + D_N^{e^+e^-} + D_S^{e^+e^-} , \quad (4)$$

where D^γ is responsible for the photons radiation and $D_N^{e^+e^-}$ and $D_S^{e^+e^-}$ describe pair production in nonsinglet (by single photon mechanism) and singlet (by double photon mechanism) channels, respectively.

The structure functions on the right-hand side of Eq. (4) satisfy the DGLAP equations [22] (see also [18]). The functions $D(z_1, L)$ and $D(z_2, L)$ is responsible for radiation of the initial and final electrons, respectively.

There exist different representations for the photonic contribution into the structure function [18, 23, 24] but here we will use the form given in [18] for D^γ , $D_N^{e^+e^-}$ and $D_S^{e^+e^-}$

$$D^\gamma(z, Q^2) = \frac{1}{2}\beta(1-z)^{\beta/2-1} \left[1 + \frac{3}{8}\beta - \frac{\beta^2}{48} \left(\frac{1}{3}L + \pi^2 - \frac{47}{8} \right) \right] - \frac{\beta}{4}(1+z) + \quad (5)$$

$$\frac{\beta^2}{32} \left[-4(1+z)\ln(1-z) - \frac{1+3z^2}{1-z} \ln z - 5 - z \right] , \quad \beta = \frac{2\alpha}{\pi}(L-1) .$$

$$D_N^{e^+e^-}(z, Q^2) = \frac{\alpha^2}{\pi^2} \left[\frac{1}{12(1-z)} (1-z - \frac{2m}{\varepsilon})^{\beta/2} (L_1 - \frac{5}{3})^2 (1+z^2 + \frac{\beta}{6}(L_1 - \frac{5}{3})) \right] \theta(1-z - \frac{2m}{\varepsilon}) , \quad (6)$$

$$D_S^{e^+e^-} = \frac{\alpha^2}{4\pi^2} L^2 \left[\frac{2(1-z^3)}{3z} + \frac{1}{2}(1-z) + (1+z)\ln z \right] \theta(1-z - \frac{2m}{\varepsilon}) . \quad (7)$$

where ε is the energy of the parent electron and $L_1 = L + 2\ln(1-z)$. Note that the above form of the structure function $D_N^{e^+e^-}$ includes effects due to real pair production only. The correction caused by the virtual pair is included in D^γ . Terms containing contribution of the order $\alpha^2 L^3$ are cancelled out in the sum $D^\gamma + D_N^{e^+e^-}$.

Instead of the photon structure function given by Eqs. (5)–(7), one can use their iterative form [23]

$$D^\gamma(z, L) = \delta(1-z) + \sum_{k=1}^{\infty} \frac{1}{k!} \left(\frac{\alpha L}{2\pi} \right)^k P_1(z)^{\otimes k} , \quad (8)$$

$$\underbrace{P_1(z) \otimes \dots \otimes P_1(z)}_k = P_1(z)^{\otimes k} , \quad P_1(z) \otimes P_1(z) = \int_z^1 P_1(t) P_1\left(\frac{z}{t}\right) \frac{dt}{t} ,$$

$$P_1(z) = \frac{1+z^2}{1-z} \theta(1-z - \Delta) + \delta(1-z) \left(2\ln \Delta + \frac{3}{2} \right) , \quad \Delta \ll 1 .$$

The iterative form (8) of D^γ does not include any effects caused by pair production. The corresponding nonsinglet part of the structure due to real and virtual pair production can be included into the iterative form of $D^\gamma(z, L)$ by replacing $\alpha L/2\pi$ on the right-hand side of Eq. (8) with the effective electromagnetic coupling

$$\frac{\alpha L}{2\pi} \rightarrow \frac{\alpha_{eff}}{2\pi} = -\frac{3}{2} \ln \left(1 - \frac{\alpha L}{3\pi} \right) \quad (9)$$

that is (within the leading accuracy) the integral of the running electromagnetic constant.

The lower limits of integration with respect to z_1 and z_2 in the master Eq.(2) can be obtained from the condition for existence of inelastic hadronic events

$$(p_1 + \tilde{q})^2 > M_{th}^2, \quad \tilde{q} = \tilde{k}_1 - \tilde{k}_2, \quad M_{th} = M + m_\pi, \quad (10)$$

where m_π is the pion mass. This constraint can be rewritten in terms of dimensionless variables as follows

$$z_1 z_2 + y - 1 - xy z_1 \leq z_2 z_{th}, \quad x = \frac{Q^2}{2p_1(k_1 - k_2)}, \quad z_{th} = \frac{M_{th}^2 - M^2}{V}, \quad (11)$$

which leads to

$$z_{2m} = \frac{1 - y + xy z_1}{z_1 - z_{th}}, \quad z_{1m} = \frac{1 + z_{th} - y}{1 - xy}.$$

The squared matrix element of the considered process in one photon exchange approximation is proportional to contraction of the leptonic and hadronic tensors. The representation (2) reflects the properties of the leptonic tensor. Therefore, it has the universal nature (because of universality of the leptonic tensor) and can be applied to processes with different final hadronic states. In particular, we can use the electron structure function method to compute RC to the elastic and deep inelastic (inclusive and semi-inclusive) electron–proton scattering cross–sections.

On the other hand, the straightforward calculations in the first order with respect to α [21, 5, 8] and the recent calculations of the leptonic current tensor in the second order [25, 26, 27, 28] for the longitudinally polarized initial electron demonstrate that in the leading approximation spin–dependent and spin–independent parts of this tensor are the same for the nonsinglet channel contribution. The latter corresponds to photon radiation and e^+e^- –pair production through the single–photon mechanism. The difference appears in the second order due to possibility of pair production in the singlet channel by double–photon mechanism [28]. Therefore, the representation (2), being slightly modified, can be used for the calculation of RC to cross–sections of different processes with a longitudinally polarized electron beam.

In our recent work [20] we applied the electron structure function method to compute RC to the ratio of the recoil proton polarizations measured at CEBAF by Jefferson Lab Hall A Collaboration [3]. The aim of this high precision experiment is the measurement of the proton electric formfactor G_E . In the present work we use this method for calculation of model–independent part of RC to the asymmetry in scattering of longitudinally polarized electrons on polarized protons at the level of per mille accuracy for elastic and deep inelastic hadronic events.

The cross–section of the scattering of the longitudinally polarized electron by the proton with given longitudinal (\parallel) or transverse (\perp) polarization for both elastic and deep inelastic events can be written as a sum of the spin–independent and spin–dependent parts

$$\frac{d\sigma(k_1, k_2, S)}{dQ^2 dy} = \frac{d\sigma(k_1, k_2)}{dQ^2 dy} + \eta \frac{d\sigma^{\parallel, \perp}(k_1, k_2, S)}{dQ^2 dy}, \quad (12)$$

where S is the 4–vector of the target proton polarization and η is the product of the electron and proton polarization degrees. Herein after we assume $\eta = 1$.

The master Eq.(2) describes the RC to the spin–independent part of the cross–section on the right-hand side of Eq. (12) and the corresponding equation for the spin–dependent part

reads

$$\frac{d\sigma^{\parallel,\perp}(k_1, k_2, S)}{dQ^2 dy} = \int_{z_{1m}}^1 dz_1 \int_{z_{2m}}^1 dz_2 D^{(p)}(z_1, L) \frac{1}{z_2^2} D(z_2, L) \frac{d^2\sigma_{hard}^{\parallel,\perp}(\tilde{k}_1, \tilde{k}_2, S)}{d\tilde{Q}^2 d\tilde{y}} , \quad (13)$$

where

$$D^{(p)} = D^\gamma + D_N^{e^+e^-} + D_S^{e^+e^-}(p) ,$$

and [28]

$$D_S^{e^+e^-}(p) = \frac{\alpha^2}{4\pi^2} L^2 \left(\frac{5(1-z)}{2} + (1+z) \ln z \right) \theta(1-z-\frac{2m}{\varepsilon}) , \quad (14)$$

describes the radiation of the initial polarized electron.

The representation is valid if radiation of collinear particles does not lead to change of polarization 4-vectors S^\parallel and S^\perp . In general it is not so [29], but in this paper we use just such polarizations which satisfy this condition (see below Eq.(18)).

The asymmetry in elastic scattering and DIS processes is defined as a ratio

$$A^{\parallel,\perp} = \frac{d\sigma^{\parallel,\perp}(k_1, k_2, S)}{d\sigma(k_1, k_2)} , \quad (15)$$

therefore RC to the asymmetry requires the knowledge of RC to both spin-independent and spin-dependent parts of the cross-section.

RC to the spin-independent part were calculated (within the electron structure function approach) in [19]. In the present work we compute the RC to spin-dependent parts for longitudinal and transverse polarizations of the target proton and longitudinally polarized electron beam. To be complete, we repeat briefly the result for unpolarized case.

3 The leading approximation

Within the leading accuracy (by taking into account the terms of the order $(\alpha L)^n$ the electron structure function can be computed, in principle, in all orders of the perturbation theory. In this approximation, we have to take the Born cross-section as a hard part on the right-hand sides of Eqs. (2) and (13).

We express the Born cross-section in terms of leptonic and hadronic tensors as follows

$$\frac{d\sigma}{dQ^2 dy} = \frac{4\pi\alpha^2(Q^2)}{VQ^4} L_{\mu\nu}^B H_{\mu\nu} , \quad (16)$$

where $\alpha(Q^2)$ is the running electromagnetic constant that accounts for the effects of vacuum polarization and

$$H_{\mu\nu} = -F_1 \tilde{g}_{\mu\nu} + \frac{F_2}{p_1 q} \tilde{p}_{1\mu} \tilde{p}_{1\nu} - i \frac{M \epsilon_{\mu\nu\lambda\rho} q_\lambda}{p_1 q} [(g_1 + g_2) S_\rho - g_2 \frac{S q}{p_1 q} p_{1\rho}] , \quad (17)$$

$$L_{\mu\nu}^B = -\frac{Q^2}{2} g_{\mu\nu} + k_{1\mu} k_{2\nu} + k_{1\nu} k_{2\mu} + i \epsilon_{\mu\nu\lambda\rho} q_\lambda k_{1\rho} , \quad \tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} , \quad \tilde{p}_{1\mu} = p_\mu - \frac{p_1 q}{q^2} q_\mu .$$

In Eqs. (17) we assume the proton and electron polarization degrees equal to 1. The spin-independent (F_1 , F_2) and spin-dependent (g_1 , g_2) proton structure functions depend on two variables

$$x' = \frac{-q^2}{2p_1q} , \quad q^2 = (p_x - p_1)^2 .$$

In Born approximation $x' = x$, but they differ in general case, when radiation of photons and electron-positron pairs is allowed.

It is convenient to parameterize the 4-vector of proton polarization in the form [8]

$$S_\mu^\parallel = \frac{2M^2 k_{1\mu} - V p_{1\mu}}{MV} , \quad S_\mu^\perp = \frac{u p_{1\mu} + V k_{2\mu} - [2u\tau + V(1-y)]k_{1\mu}}{\sqrt{-uV^2(1-y) - u^2M^2}} , \quad (18)$$

where $u = -Q^2$, $\tau = M^2/V$.

One can verify that the 4-vector S^\parallel in the laboratory system has components $(0, \vec{n})$, where 3-vector \vec{n} has orientation of the initial electron 3-momentum \vec{k}_1 . One can verify also that $S^\perp S^\parallel = 0$ and in the laboratory system

$$S^\perp = (0, \vec{n}_\perp) , \quad \vec{n}_\perp^2 = 1 , \quad \vec{n}\vec{n}_\perp = 0 ,$$

where 3-vector \vec{n}_\perp belongs to the plane (\vec{k}_1, \vec{k}_2) .

As normalization is chosen, the elastic limit ($p_x^2 = M^2$) can be reached by a simple substitution in the hadronic tensor

$$F_1(x', q^2) \rightarrow \frac{1}{2}\delta(1-x')G_M^2(q^2) , \quad F_2(x', q^2) \rightarrow \delta(1-x')\frac{G_E^2(q^2) + \lambda G_M^2(q^2)}{1+\lambda} , \quad (19)$$

$$g_1(x', q^2) \rightarrow \frac{1}{2}\delta(1-x')\{G_M(q^2)G_E(q^2) + \frac{\lambda}{1+\lambda}[G_M(q^2) - G_E(q^2)]G_M(q^2)\} ,$$

$$g_2(x', q^2) \rightarrow -\frac{1}{2}\delta(1-x')\frac{\lambda}{1+\lambda}[G_M(q^2) - G_E(q^2)]G_M(q^2) , \quad \lambda = -\frac{q^2}{4M^2}$$

where G_M and G_E are magnetic and electric proton formfactors.

A simple calculation gives the spin-independent and spin-dependent parts of the well known Born cross-section in the form

$$\frac{d\sigma^B}{dQ^2 dy} = \frac{4\pi\alpha^2(Q^2)}{Q^4 y} [(1-y-xy\tau)F_2(x, Q^2) + xy^2 F_1(x, Q^2)] , \quad (20)$$

$$\frac{d\sigma_\parallel^B}{dQ^2 dy} = \frac{8\pi\alpha^2(Q^2)}{V^2 y} [(\tau - \frac{2-y}{2xy})g_1(x, Q^2) + \frac{2\tau}{y}g_2(x, Q^2)] , \quad (21)$$

$$\frac{d\sigma_\perp^B}{dQ^2 dy} = -\frac{8\pi\alpha^2(Q^2)}{V^2 y} \sqrt{\frac{M^2}{Q^2}(1-y-xy\tau)} [g_1(x, Q^2) + \frac{2}{y}g_2(x, Q^2)] . \quad (22)$$

Thus, within the leading accuracy, the radiatively corrected cross-section of the process (1) is defined by Eq.(2) (for its spin-independent part) with (20) as a hard part of the cross-section and by Eq.(13) (for its spin-dependent part) with (21) or (22) as a hard part.

It is useful to extract the first order correction to Born approximation, as defined by master equation (2). For this purpose, we can use the iterative form of the photon structure function D^γ with $L \rightarrow L - 1$ and

$$\Delta \rightarrow \Delta_1 = \frac{2(\Delta\varepsilon)}{\sqrt{V}(1-xy)}\sqrt{\tau+z_+}, \quad z_+ = y(1-x), \quad \frac{2(\Delta\varepsilon)}{\sqrt{V}} \ll 1$$

for $D(z_1, L)$ and

$$\Delta \rightarrow \Delta_2 = \frac{2(\Delta\varepsilon)}{\sqrt{V}(1-z_+)}\sqrt{\tau+z_+}$$

for $D(z_2, L)$, where $(\Delta\varepsilon)$ is the minimal energy of hard collinear photon in the special system ($\vec{k}_1 - \vec{k}_2 + \vec{p}_1 = 0$). Straightforward calculations yield the following expression

$$\begin{aligned} \frac{d\sigma^{(1)}(k_1, k_2)}{dQ^2 dy} &= \frac{\alpha(L-1)}{2\pi} \left\{ \frac{d\sigma^{(B)}(k_1, k_2)}{dQ^2 dy} \left[3 + 2 \ln \frac{4(\Delta\varepsilon)^2(z_+ + \tau)}{V(1-z_+)(1-xy)} \right] + \right. \\ &\quad \left. \int_{z_{th}}^{z_+ - \rho} dz \left[\frac{1+z_1^2}{(1-xy)(1-z_1)} \frac{d\sigma^{(B)}(z_1 k_1, k_2)}{dQ_t^2 dy_t} + \frac{1+z_2^2}{(1-z_+)(1-z_2)} \frac{d\sigma^{(B)}(k_1, k_2/z_2)}{dQ_s^2 dy_s} \right] \right\}, \end{aligned} \quad (23)$$

where

$$\begin{aligned} z &= \frac{M_x^2 - M^2}{V}, \quad z_1 = \frac{1-y+z}{1-xy}, \quad z_2 = \frac{1-z_+}{1-z}, \quad \rho = \frac{2(\Delta\varepsilon)}{\sqrt{V}}\sqrt{\tau+z_+}, \\ Q_t^2 &= -q_t^2 = z_1 Q^2, \quad Q_s^2 = -q_s^2 = \frac{Q^2}{z_2}, \quad y_{t,s} = 1 - \frac{1-y}{z_{1,2}}. \end{aligned}$$

Similar equations can be derived for the first order correction to the spin-dependent part of the cross-section for both longitudinal and transverse polarizations of the target proton.

4 DIS cross-section beyond the leading accuracy

To go beyond the leading accuracy we have to improve the expressions for hard parts of the cross-sections in master equations (2) and (13) to include effects caused by radiation of a hard noncollinear photon. (In principle, we can improve also the expression for D -function to take into account collinear next-to-leading effects in the second order of perturbation theory. The essential part of these effects is included in our D -functions due to replacement $L \rightarrow L - 1$. The rest can be written by using the results of corresponding calculations for double photon emission [25, 26, 27, 30], pair production [28, 31, 32], one loop corrected Compton tensor [25, 26, 33] and virtual correction [34]. But here we restrict ourselves to D -functions given above in Eqs. (5), (6), (7) and (14)).

To compute the improved hard cross-section, one has to find the full first order RC to the cross-section of the process (1) and subtract from it (to get rid of the double counting) its leading part that (for unpolarized case) is defined by Eq. (23). Therefore, the improved hard part can be written as

$$\frac{d\sigma_{hard}}{dQ^2 dy} = \frac{d\sigma^B}{dQ^2 dy} + \frac{d\sigma^{(S+V)}}{dQ^2 dy} + \frac{d\sigma^H}{dQ^2 dy} - \frac{d\sigma^{(1)}}{dQ^2 dy}, \quad (24)$$

where $d\sigma^{(S+V)}$ is a correction to the cross-section of the process (1) due to virtual and soft photon emission and $d\sigma^H$ is a cross-section of the radiative process

$$e^-(k_1) + P(p_1) \rightarrow e^-(k_2) + \gamma(k) + X(p_x) . \quad (25)$$

The virtual and soft corrections are factorized in the same form for both polarized and unpolarized cases [19] and can be written as

$$\frac{d\sigma^{(S+V)}}{dQ^2 dy} = \frac{d\sigma^B}{dQ^2 dy} \left[1 + \frac{\alpha}{2\pi} \left(\delta + (L-1) \left(3 + 2 \ln \frac{\rho^2}{(1-xy)(1-z_+)} \right) \right) \right] , \quad (26)$$

$$\delta = -1 - \frac{\pi^2}{3} - 2f\left(\frac{1-y-xy\tau}{(1-xy)(1-z_+)}\right) - \ln^2 \frac{1-xy}{1-z_+} , \quad f(x) = \int_0^x \frac{dt}{t} \ln(1-t) .$$

To calculate the cross-section of the radiative process (25), we use the corresponding leptonic tensor in the form

$$L_{\mu\nu}^\gamma = \frac{\alpha}{4\pi^2} (L_{\mu\nu}^{H(un)} + L_{\mu\nu}^H) \frac{d^3k}{\omega} , \quad L_{\mu\nu}^H = 2i\varepsilon_{\mu\nu\lambda\rho} q_\lambda (k_{1\rho} R_t + k_{2\rho} R_s) , \quad (27)$$

$$R_t = \frac{u+t}{st} - 2m^2 \left(\frac{1}{s^2} + \frac{1}{t^2} \right) , \quad R_s = \frac{u+s}{st} - 2m^2 \frac{s_t}{ut^2} , \quad s_t = \frac{-u(u+Vy-Vz)}{u+V} ,$$

where ω is the energy of radiated photon, $L_{\mu\nu}^{H(un)}$ is the leptonic tensor for unpolarized particles, see Ref. [33], and we use the following notation for kinematic invariants

$$s = 2kk_2 , \quad t = -2kk_1 , \quad q^2 = u + s + t .$$

The result for unpolarized case was derived in [19], and here we rewrite it in terms of our standard notation

$$\frac{d\sigma_{hard}}{dQ^2 dy} = \frac{d\sigma^B}{dQ^2 dy} \left(1 + \frac{\alpha}{2\pi} \delta \right) + \frac{\alpha}{VQ^2} \int_{z_{th}}^{z_+} dz \left\{ \frac{1-r_1}{1-xy} \hat{P}_t N - \frac{1-r_2}{1-z_+} \hat{P}_s N + \int_{r_-}^{r_+} dr \frac{2W}{\sqrt{y^2 + 4xy\tau}} + \right. \quad (28)$$

$$\left. + P \int_{r_-}^{r_+} \frac{dr}{1-r} \left[\frac{1-\hat{P}_t}{|r-r_1|} \left(\frac{(1+r^2)N}{1-xy} + (r_1-r)T_t \right) - \frac{1-\hat{P}_s}{|r-r_2|} \left(\frac{(1+r^2)N}{1-z_+} + (r_2-r)T_s \right) \right] \right\} \frac{\alpha^2(rQ^2)}{r^2} ,$$

where $r = -q^2/Q^2$ and the limits of the integration respect to r are

$$r_\pm(z) = \frac{1}{2xy(\tau+z_+)} [2xy(\tau+z) + (z_+-z)(y \pm \sqrt{y^2 + 4xy\tau})] .$$

Here we used the following notation

$$N = 2F_1(x', r) + \frac{2x'}{rxy} \left(\frac{1-y}{xy} - \tau \right) F_2(x', r) , \quad W = 2F_1(x', r) - \frac{2x'\tau}{rxy} F_2(x', r) ,$$

$$T_t = -\frac{2x'[1-r(1-y)]}{x^2 y^2 r} F_2(x', r) , \quad T_s = -\frac{2x'(1-y-r)}{x^2 y^2 r} F_2(x', r) , \quad (29)$$

$$r_1 = \frac{1-y+z}{1-xy} , \quad r_2 = \frac{1-z}{1-z_+} , \quad x' = \frac{xyr}{xyr+z} .$$

The action of the operators \hat{P}_t and \hat{P}_s is defined as follows

$$\hat{P}_t f(r, x') = f(r_1, x_t) , \quad \hat{P}_s f(r, x') = f(r_2, x_s) , \quad x_t = \frac{xyr_1}{xyr_1+z} , \quad x_s = \frac{xyr_2}{xyr_2+z} .$$

Note that the quantity $r_1(r_2)$ coincides with $z_1(1/z_2)$ for radiation of a single collinear photon.

The hard cross-section (29) has neither collinear nor infrared singularities. The different terms on the right-hand side of Eq. (29) have singularities at $r = r_1$, $r = r_2$ and $r = 1$. Singularities at first two points are collinear and at third one is unphysical that arises at integration. Collinear singularities vanish due to action of operators \hat{P}_t and \hat{P}_s on the terms containing N . The unphysical singularity cancels because in the limiting case $r \rightarrow 1$ we have

$$\frac{r_2 - r}{|r_2 - r|} = 1 , \quad \frac{r_1 - r}{|r_1 - r|} = -1 , \quad T_t + T_s = 0 .$$

Let us consider the spin-dependent part of hard cross-section in more details. The contraction of the spin-dependent parts of leptonic and hadronic tensors can be written as

$$\begin{aligned} L_{\mu\nu}^H H_{\mu\nu}^{\parallel,\perp} &= -U^{\parallel,\perp} C^{\parallel,\perp} \frac{x'}{q^2} , \quad U^{\parallel} = 1 , \quad U^{\perp} = \frac{1}{V} \sqrt{\frac{M^2}{Q^2} (1-y-xy\tau)^{-1}} , \\ C^{\parallel,\perp} &= \tilde{2}W^{\parallel,\perp} + \left[\frac{u^2 + q_t^4}{t(q_t^2 - u)} - \frac{2m^2}{ut^2} (u^2 + q_t^2 s_t) \right] \hat{P}_t \tilde{N}_t^{\parallel,\perp} + \left[\frac{u^2 + q_s^4}{s(q_s^2 - u)} - \frac{2m^2}{s^2} q_s^2 \right] \hat{P}_s \tilde{N}_s^{\parallel,\perp} + \\ &\quad \frac{1 - \hat{P}_t (u^2 + q^4) \tilde{N}_t^{\parallel,\perp} + 2q^2 (q_t^2 - q^2) \tilde{T}_t^{\parallel,\perp}}{t} + \frac{1 - \hat{P}_s (u^2 + q^4) \tilde{N}_s^{\parallel,\perp} + 2u (q_s^2 - q^2) \tilde{T}_s^{\parallel,\perp}}{s} , \\ q_t^2 &= \frac{uV(1-y+z)}{u+V} , \quad q_s^2 = \frac{uV(1-z)}{V(1-y)-u} . \end{aligned} \quad (30)$$

For the case of longitudinal polarization of the target proton we have

$$\begin{aligned} \tilde{W}^{\parallel} &= 4\tau[yx'Vg_2 - (q^2 + u)g_1] , \quad \tilde{N}_s^{\parallel} = 2(2q^2\tau + 2V + \frac{q^2}{x'})g_1 - 8\tau x'Vg_2 , \\ \tilde{N}_t^{\parallel} &= 2[2u\tau + q^2(\frac{2V}{u} + \frac{1}{x'})]g_1 - 8\tau x'Vg_2 , \quad \tilde{T}_s^{\parallel} = \frac{2uV(z-1)}{q_s^2}(g_1 - 2\tau x'g_2) , \\ \tilde{T}_t^{\parallel} &= 2(u+V)(\frac{q^2}{u}g_1 - 2\tau x'g_2) . \end{aligned} \quad (31)$$

The corresponding quantities for the case of transverse polarization of the target proton read

$$\begin{aligned} \tilde{W}^{\perp} &= 2(2u\tau - Vy)[(q^2 + u)g_1 - x'yVg_2] , \\ \tilde{N}_s^{\perp} &= 2[-uV - q^2(2u\tau + V(1-y) + \frac{u}{x'})](g_1 - \frac{2x'V}{q^2}g_2) , \\ \tilde{N}_t^{\perp} &= 2[-q^2V - u(2u\tau + V(1-y) + \frac{q^2}{x'})](g_1 - \frac{2x'V}{u}g_2) , \end{aligned} \quad (32)$$

$$\begin{aligned}\tilde{T}_t^\perp &= 2(u+V)[-q^2 g_1 + \frac{x'V}{u}(q^2 + u(1-y + \frac{2u\tau}{V}))g_2] , \\ \tilde{T}_s^\perp &= 2\frac{uV(1-z)}{q_s^2}[ug_1 - \frac{x'V}{q^2}(u + q^2(1-y + \frac{2u\tau}{V}))g_2] .\end{aligned}$$

The action of operators \hat{P}_t and \hat{P}_s in the expressions for $C^{\parallel,\perp}$ can be understood if we write

$$r = \frac{q^2}{u} , \quad r_1 = \frac{q_t^2}{u} , \quad r_2 = \frac{q_s^2}{u} , \quad x' = \frac{q^2}{q^2 - Vz} .$$

The cross-section of radiative process (25) can be written in terms of the quantities $C^{\parallel,\perp}$ as follows

$$\frac{d\sigma_{\parallel,\perp}^H}{dQ^2 dy} = -\frac{2\alpha}{V} U^{\parallel,\perp} C^{\parallel,\perp} \frac{x'\alpha^2(-q^2)}{q^6} \omega d\omega d\cos\theta_k \frac{d\phi}{2\pi} , \quad (33)$$

where θ_k and ϕ are polar and azimuth angles of photon in the special system with Z -axis along the direction of the target proton 3-momentum \vec{p}_1 , provided \vec{k}_1 and \vec{k}_2 are within XZ plane.

Integration of (33) with respect to photon variables can be done in full analogy with unpolarized case as described in [19] (see also [20]). The result can be written in the following form

$$\begin{aligned}\frac{d\sigma_{\parallel,\perp}^H}{dQ^2 dy} &= -\frac{\alpha}{V} U^{\parallel,\perp} \int_{z_{th}}^{z_+} dz \left\{ \frac{V}{u+V} [q_t^2 - u - \frac{u^2 + q_t^4}{q_t^2 - u} (L-1)] \hat{P}_t \tilde{N}_t^{\parallel,\perp} + \frac{V}{V(1-y) - u} [q_s^2 - u + \right. \\ &\quad \left. \frac{u^2 + q_s^4}{q_s^2 - u} (L-1)] \hat{P}_s \tilde{N}_s^{\parallel,\perp} + P \int_{q_-^2}^{q_+^2} \frac{dq^2}{q^2 - u} \left[\frac{V(1 - \hat{P}_s)}{(V(1-y) - u)|q^2 - q_s^2|} ((u^2 + q^4) \tilde{N}_s^{\parallel,\perp} + \right. \right. \\ &\quad \left. \left. 2u(q_s^2 - q^2) \tilde{T}_s^{\parallel,\perp}) - \frac{V(1 - \hat{P}_t)}{(V+u)|q^2 - q_t^2|} ((u^2 + q^4) \tilde{N}_t^{\parallel,\perp} + 2q^2(q_t^2 - q^2) \tilde{T}_t^{\parallel,\perp}) \right] + \right. \\ &\quad \left. \int_{q_-^2}^{q_+^2} \frac{dq^2}{\sqrt{y^2 + 4xy\tau}} \tilde{2}W^{\parallel,\perp} \right\} \frac{x'\alpha^2(-q^2)}{q^6} , \quad q_\pm^2 = ur_\mp .\end{aligned} \quad (34)$$

To derive the hard cross-section for the polarized case we have to add (26) and (34) without their leading contributions, which are proportional to $L-1$ and sum up to $d\sigma_{\parallel,\perp}^{(1)}/dQ^2 dy$. The result reads

$$\begin{aligned}\frac{d\sigma_{hard}^{\parallel,\perp}}{dQ^2 dy} &= \frac{d\sigma_{\parallel,\perp}^B}{dQ^2 dy} (1 + \frac{\alpha}{2\pi} \delta) + \frac{\alpha}{Q^4} \tilde{U}^{\parallel,\perp} \int_{z_{th}}^{z_+} dz \left\{ \frac{1-r_1}{1-xy} \hat{P}_t N_t^{\parallel,\perp} + \frac{1-r_2}{1-z_+} \hat{P}_s N_s^{\parallel,\perp} + \right. \\ &\quad \left. + P \int_{r_-}^{r_+} \frac{dr}{1-r} \left[\frac{1 - \hat{P}_s}{|r - r_2|(1 - z_+)} ((1 + r^2) N_s^{\parallel,\perp} + \frac{2(r_2 - r)}{r_2} T_s^{\parallel,\perp}) - \frac{1 - \hat{P}_t}{|r - r_1|} \left(\frac{(1 + r^2) N_t^{\parallel,\perp}}{1 - xy} + \right. \right. \\ &\quad \left. \left. 2r(r_1 - r) T_t^{\parallel,\perp} \right) \right] + \int_{r_-}^{r_+} dr \frac{2W^{\parallel,\perp}}{\sqrt{y^2 + 4xy\tau}} \right\} \frac{x'\alpha^2(Q^2 r)}{r^3} ,\end{aligned} \quad (35)$$

where

$$\begin{aligned}
\tilde{U}^{\parallel} &= 1, \quad \tilde{U}^{\perp} = \sqrt{\frac{M^2}{Q^2}(1-y-xy\tau)^{-1}}, \\
W^{\parallel} &= 4y\tau W, \quad W^{\perp} = 2y^2(1+2x\tau)W, \quad W = (1+r)xg_1 + x'g_2, \\
N_t^{\parallel} &= 2[2r-z-xy(r+2\tau)]g_1 - 8x'\tau g_2, \quad N_s^{\parallel} = 2[2-z-xyr(1+2\tau)]g_1 - 8x'\tau g_2, \\
N_t^{\perp} &= 2[1-y-z+r-xy(r+2\tau)](xyg_1+2x'g_2), \quad N_s^{\perp} = 2[1-y+\frac{1-z}{r}-xy(1+2\tau)](xyrg_1+2x'g_2), \\
T_t^{\parallel} &= 2rg_1 - 4x'\tau g_2, \quad T_s^{\parallel} = 2(z-1)(g_1 - 2x'\tau g_2), \\
T_t^{\perp} &= 2xyrg_1 + 2x'(1-y+r-2xy\tau)g_2, \quad T_s^{\perp} = 2(z-1)[xyg_1 + x'(1-y+\frac{1}{r}-2xy\tau)g_2].
\end{aligned}$$

The polarized hard cross-section defined by Eq. (35) is free from collinear singularities due to action of operators $1 - \hat{P}_t$ and $1 - \hat{P}_s$. The unphysical singularity at $r = 1$ on the right-hand side of Eq. (35) cancels because in this limit

$$T_t^{\parallel,\perp} = \frac{1}{z-1} T_s^{\parallel,\perp}.$$

Note that radiation of photon at large angles by the initial and final electrons increases the region of variation for quantity r in (35), because for collinear radiation $r_1 < r < r_2$ and now $r_- < r_1$ and $r_+ > r_2$. It may be important if the hadron structure functions are large in these additional regions.

5 Hard cross-section for elastic hadronic events

To describe the hard cross-section for elastic hadronic events we use the replacement defined by (19) in Eqs. (28) and (35). For Born cross-sections which enter in this equations, see Eqs.(21)–(23). The function $\delta(1-x')$ is used to do the integration with respect to inelasticity z

$$\int dz \delta(1-x') = xy r. \quad (36)$$

The final result for unpolarized case reads (we do not introduce special notation for the elastic cross-section)

$$\begin{aligned}
\frac{d\sigma_{hard}}{dQ^2 dy} &= \frac{d\sigma^B}{dQ^2 dy} (1 + \frac{\alpha}{2\pi} \delta) + \frac{\alpha}{V^2} \left\{ \frac{1-r_1}{1-xy} \hat{P}_t N - \frac{1-r_2}{1-z_+} \hat{P}_s N + \int_{r_-}^{r_+} dr \frac{2W}{\sqrt{y^2 + 4xy\tau}} + \right. \\
&\quad \left. P \int_{r_-}^{r_+} \frac{dr}{1-r} \left[\frac{1-\hat{P}_t}{|r-r_1|} \left(\frac{1+r^2}{1-xy} N + (r_1-r) T_t \right) - \frac{1-\hat{P}_s}{|r-r_2|} \left(\frac{1+r^2}{1-z_+} N + (r_2-r) T_s \right) \right] \right\} \frac{\alpha^2(Q^2 r)}{r},
\end{aligned} \quad (37)$$

where

$$N = G_M^2 + \frac{2}{xyr} \left(\frac{1-y}{xy} - \tau \right) \frac{G_E^2 + \lambda G_M^2}{1+\lambda}, \quad W = G_M^2 - \frac{2\tau}{xyr} \frac{G_E^2 + \lambda G_M^2}{1+\lambda},$$

$$T_t = -\frac{2}{x^2 y^2 r} [1 - r(1 - y)] \frac{G_E^2 + \lambda G_M^2}{1 + \lambda}, \quad T_s = -\frac{2}{x^2 y^2 r} (1 - r - y) \frac{G_E^2 + \lambda G_M^2}{1 + \lambda}.$$

The Born cross-section on the right-hand side of Eq.(37) is defined as

$$\frac{d\sigma^B}{dQ^2 dy} = \frac{4\pi\alpha^2(Q^2)}{V^2} \left[\frac{1}{2} G_M^2 + [1 - y(1 + \tau)] \frac{G_E^2 + \lambda G_M^2}{y^2(1 + \lambda)} \right] \delta(y - \frac{Q^2}{V}). \quad (38)$$

When writing this last equation we take into account that

$$\delta(1 - x) = y \delta(y - \frac{Q^2}{V}).$$

The spin-dependent hard cross-section for elastic hadronic events can be written in the form very similar to (37)

$$\begin{aligned} \frac{d\sigma_{hard}^{\parallel,\perp}}{dQ^2 dy} = & \frac{d\sigma_{\parallel,\perp}^B}{dQ^2 dy} (1 + \frac{\alpha}{2\pi} \delta) + \frac{\alpha}{V} \tilde{U}^{\parallel,\perp} \left\{ \frac{1 - r_1}{1 - xy} \hat{P}_t N_t^{\parallel,\perp} + \frac{1 - r_2}{1 - z_+} \hat{P}_s N_s^{\parallel,\perp} + \int_{r_-}^{r_+} dr \frac{W^{\parallel,\perp}}{\sqrt{y^2 + 4xy\tau}} + \right. \\ & P \int_{r_-}^{r_+} \frac{dr}{1 - r} \left[-\frac{1 - \hat{P}_t}{|r - r_1|} \left(\frac{1 + r^2}{1 - xy} N_t^{\parallel,\perp} + 2r(r_1 - r) T_t^{\parallel,\perp} \right) + \right. \\ & \left. \left. \frac{1 - \hat{P}_s}{|r - r_2|(1 - z_+)} \left((1 + r^2) N_s^{\parallel,\perp} + \frac{2(r_2 - r)}{r_2} T_s^{\parallel,\perp} \right) \right] \right\} \frac{\alpha^2(Q^2 r)}{(4M^2 + Q^2 r) r^2}, \end{aligned} \quad (39)$$

where

$$W^{\parallel} = 4y\tau W, \quad W^{\perp} = 2y^2(1 + 2x\tau)W, \quad W = r[x(1 + r) - 1]G_M^2 + [r + \frac{4\tau}{y}(1 + r)]G_M G_E,$$

$$N_t^{\parallel} = r(2\tau + r)(2 - xy)G_M^2 + 8\tau[r(\frac{1}{xy} - 1) - \tau]G_M G_E,$$

$$N_s^{\parallel} = r(2\tau + 1)(2 - xy r)G_M^2 + 8\tau[\frac{1}{xy} - r(1 + \tau)]G_M G_E,$$

$$N_t^{\perp} = [1 - y + r - xy(r + 2\tau)][-r(2 - xy)G_M^2 + 2(r + 2\tau)G_M G_E],$$

$$N_s^{\perp} = [1 - y + \frac{1}{r} - xy(1 + 2\tau)][-r(2 - xy r)G_M^2 + 2r(1 + 2\tau)G_M G_E],$$

$$T_t^{\parallel} = r[(r + 2\tau)G_M^2 + 2\tau(\frac{2}{xy} - 1)G_M G_E], \quad T_s^{\parallel} = -r(1 + 2\tau)G_M^2 - 2\tau(\frac{2}{xy} - r)G_M G_E,$$

$$T_t^{\perp} = r\{-[r(1 - xy) + 1 - y - 2xy\tau]G_M^2 + [1 - y - 2xy\tau + r + 4\tau]G_M G_E\},$$

$$T_s^{\perp} = r[\frac{1}{r} - xy(1 + 2\tau) + 1 - y]G_M^2 - [2\tau(2 - xy r) + 1 + r(1 - y)]G_M G_E.$$

Note that the argument of electromagnetic formfactors in Eqs.(37) and (39) is $-Q^2 r$.

The Born cross-sections on the right-hand side of Eq (39) have the following form

$$\frac{d\sigma_{\parallel}^B}{dQ^2 dy} = \frac{4\pi\alpha^2(Q^2)}{V(4M^2 + Q^2)} \left[4\tau(1 + \tau - \frac{1}{y})G_M G_E - (1 + 2\tau)(1 - \frac{y}{2})G_M^2 \right] \delta(y - \frac{Q^2}{V}), \quad (40)$$

for the longitudinal polarization of the target proton and

$$\frac{d\sigma_{\perp}^B}{dQ^2 dy} = \frac{8\pi\alpha^2(Q^2)}{V(4M^2 + Q^2)} \sqrt{\frac{M^2}{Q^2}} [1 - y(1 + \tau)] \left[(1 - \frac{y}{2})G_M^2 - (1 + 2\tau)G_M G_E \right] \delta(y - \frac{Q^2}{V}), \quad (41)$$

for the transverse one. The argument of formfactors in (40), (41) is $-Q^2$.

6 Numerical estimations

The formulae obtained in the last section include some operators which emphasize the physical meaning of made transformations. However they are not convenient to numerical analysis. Here we present a unified version of the formulae without any operators. For example, the symbol P is explicitly treated as

$$P \int_{r_-}^{r_+} \frac{dr}{1-r} F(r) = \int_{r_-}^{r_+} \frac{dr}{1-r} (F(r) - F(1)) + F(1) \log \frac{1-r_-}{r_+-1}$$

So the formula reads

$$\begin{aligned} \frac{d\sigma_{hard}^i}{dQ^2 dy} &= \frac{d\sigma_i^B}{dQ^2 dy} \left(1 + \frac{\alpha}{2\pi} \delta\right) + \alpha U_i \int_{z_{th}}^{z_+} dz \left(\left\{ L_1^i N_i(r_1) + L_2^i N_i(r_2) \right\} + \int_{r_-}^{r_+} dr \left\{ W_i + T_i + \right. \right. \\ &\quad \left. \left. + \frac{1}{1-r} \left[N_i(r_1) - N_i(r_2) + \frac{1-r_1}{|r-r_1|} [N_i(r) - N_i(r_1)] + \frac{1-r_2}{|r-r_2|} [N_i(r) - N_i(r_2)] \right] \right\} \right) \end{aligned} \quad (42)$$

where

$$L_{1,2}^i = \mp b_i \frac{(1-r_{1,2})^2}{1+r_{1,2}^2} \mp \log \frac{1-r_-}{r_+-1}, \quad b_u = -1, \quad b_{l,t} = 1.$$

The index i runs over all polarization states ($i = u, l, t$). The functions $N_i(r)$ and T_i read

$$N_i(r) = \frac{1+r^2}{z_+-z} N_i \frac{x'\alpha^2}{r^3}, \quad T_i = \begin{cases} \pm \frac{T_{i1}}{1-r} \frac{x'\alpha^2}{r^3} & r < r_1; \quad r > r_2 \\ T_{i2} \frac{x'\alpha^2}{r^3} & r_1 < r < r_2 \end{cases}$$

the pole $r = 1$ can be reached only in the region $r_1 < r < r_2$, so there is no singularity in terms with T_{i1} . For T_{i2} this pole is explicitly canceled:

$$T_{u2} = \frac{2(2-y)F_2}{x^2 y^2}, \quad T_{l2} = -4(1+r)g_1 + 8x'\tau g_2, \quad T_{t2} = -4(1+r)xyg_1 - 4x'(r + \frac{1}{r} + 2 - y - 2xy\tau)g_2.$$

For unpolarized case $N_u = rN/x'$ with n from (28). For other cases they are

$$\begin{aligned} N_l &= 2 \left[-1 - r + \frac{y(1+2x\tau)[1-z+r(1-xy)]}{2-y} \right] g_1 + 8x'\tau g_2, \\ N_t &= -\frac{4[1-z+r(1-xy)]}{r(2-y)} \left[xyr(1-y-xy\tau)g_1 + x'(1-y+z+r(1-y+xy))g_2 \right] + \\ &\quad + 4x'y(1+2x\tau)g_2, \\ T_{u1} &= -\frac{2(1+r)F_2}{x^2 y}, \\ T_{l1} &= 4 \frac{y(1+r^2)(1+2x\tau)}{2-y} g_1 + 8x'(1+r)\tau g_2, \\ T_{t1} &= 4 \left\{ \frac{1+r^2}{2-y} \left[-2xy(1-y-xy\tau)g_1 + (y-2z+yr(1-2x))\frac{x'}{r}g_2 \right] \right. \\ &\quad \left. + x'y(1+2x\tau)(1+r)g_2 \right\} \end{aligned}$$

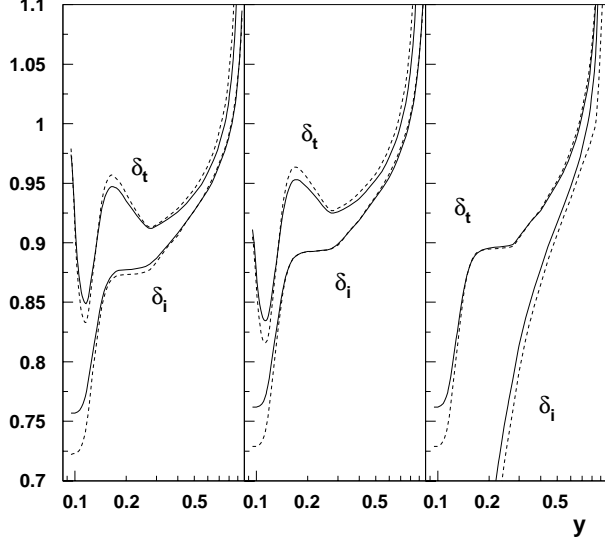


Figure 1: Radiative correction to unpolarized and polarized (both longitudinal and transverse) parts of cross section for kinematics close to JLab experiments, $V=10 \text{ GeV}^2$, $x=0.5$.

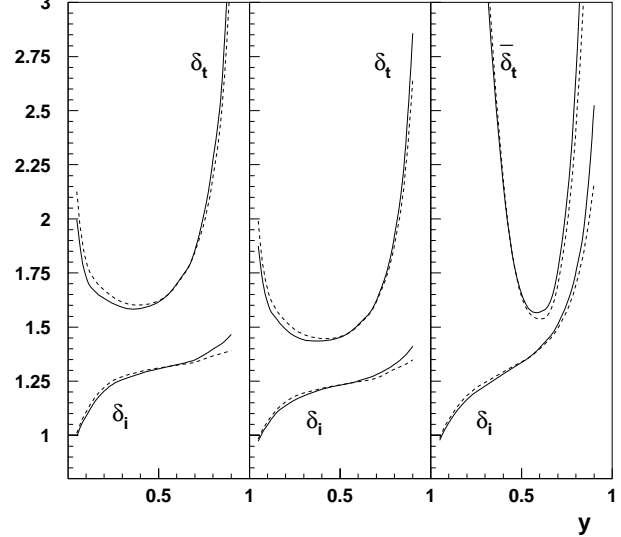


Figure 2: Radiative correction to unpolarized and polarized (both longitudinal and transverse) parts of cross section for kinematics close to HERMES experiment, $V=50 \text{ GeV}^2$, $x=0.1$. $\bar{\delta}_t = -\delta_t$

and

$$W_u = \frac{2W}{\sqrt{y^2 + 4xy\tau}} \frac{\alpha^2}{r^2}, \quad W_{l,t} = \frac{2W^{\parallel,\perp}}{\sqrt{y^2 + 4xy\tau}} \frac{x\alpha^2}{r^3}$$

$$U_u = \frac{1}{VQ^2}, \quad U_{l,t} = \frac{\tilde{U}^{\parallel,\perp}}{Q^4}$$

For elastic case the same formulae can be kept. Only the formulae (19) and (36) are needed here. So for elastic case one has to substitute

$$\int dz \rightarrow xyr,$$

setting $x' = 1$, $z = 0$ and structure functions in accordance with (19).

It is believed that the formulae obtained within the presented formalism are not convenient for numerical analysis. There are two reasons for such an opinion. First, the electron structure function in the form (5,6) has very sharp peak for z going to unity. Secondly, due to appearing of absolute values in denominators, the integrand cannot be a continuous function of the integration variables. It produces obstacles for numerical analysis if it is carried out in traditional style based on adaptive methods of numerical integration, which is used in such programs as TERAD/HECTOR [36] or POLRAD [9]. However, it is possible to perform numerical analysis if instead of adaptive integration we use Monte Carlo integration while extracting the regions with sharp peaks into separate integration subregions. Based on these ideas we developed Fortran code ESFRAD¹ which allows one to perform the numerical analysis without any serious difficulties.

We considered two radiative processes. In the first case, continuum of hadrons is produced, while in the second case the proton remains in the ground state. Both of the considered effects contribute to the experimentally observed cross section² of DIS. They are usually called

¹Electron Structure a Function method for RAdiative corrections

²Here and below we mean double differential cross section $\sigma = d\sigma/dy dQ^2$

radiative tails from the continuous spectrum and the elastic peak or simply inelastic and elastic radiative tails. Below we study the contributions of the tails numerically within kinematical conditions of the current experiments on DIS.

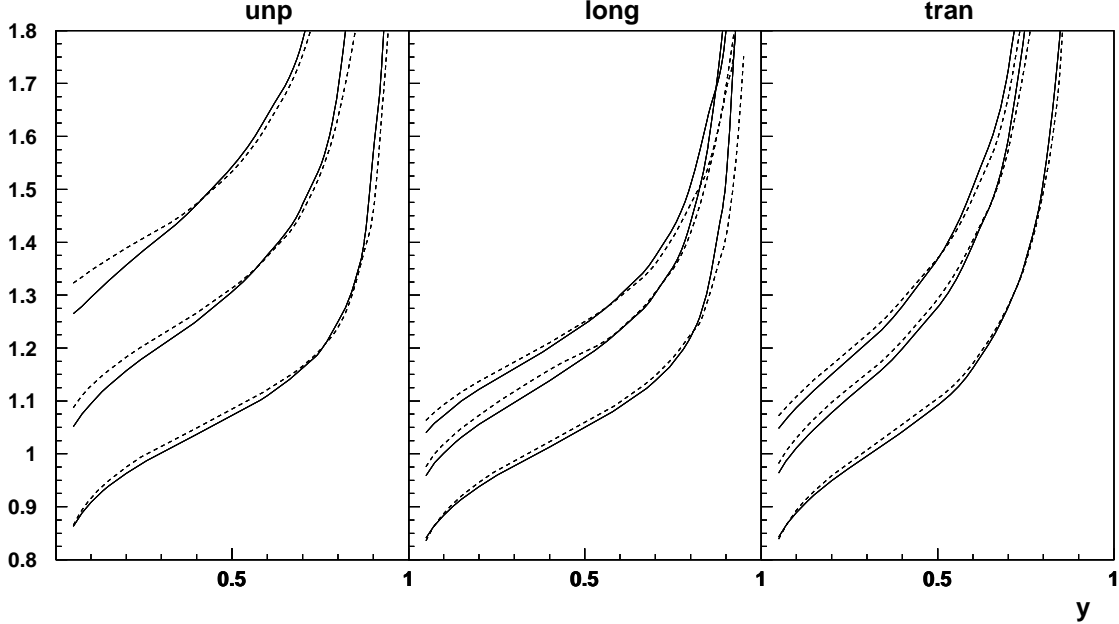


Figure 3: One loop and total radiative correction (dashed and solid lines) for collider kinematics (HERA); $V=10^5 \text{ GeV}^2$. Lines from top to bottom correspond to different values of $x=0.001$, 0.01 and 0.1

We take three typical values of V equal to 10, 50 and 10000 GeV^2 . They correspond to JLab, HERMES and HERA measurements. Figures 1, 2 and 3 give the radiative correction factor for all polarization states (unpolarized, longitudinal or transverse)

$$\delta_{i,t} = \frac{\sigma^{obs}}{\sigma^B}. \quad (43)$$

The observed double differential cross section is given by the master formulae (2,13), while the Born cross section is calculated as (20,21,22). Both elastic and inelastic contributions have to be taken for σ_{hard} . In this case we obtain the total RC factor (δ_t). Subscripts i, t correspond to the cases when elastic radiative tail is included into total correction (δ_t) or inelastic radiative tail contributes only (δ_i). The elastic radiative tail optionally may not be included because sometimes there exist experimental methods to separate this contribution. We note that for HERA kinematics we do not include it because it is usually separated experimentally. Also we can extract a one-loop contribution in order to study the effect of higher order correction. The observed cross section in this case is defined by the sum of the cross sections defined in eqs. (23) and (42). We note that it can provide an additional cross check by comparison with POLRAD.

We use rather simple models for spin-averaged and spin-dependent structure functions. It allows us not to mix pure radiative effects, which are of interest, with influence of hadron structure functions. Specifically, we use the so-called D8 model for spin-average SF [35] (see

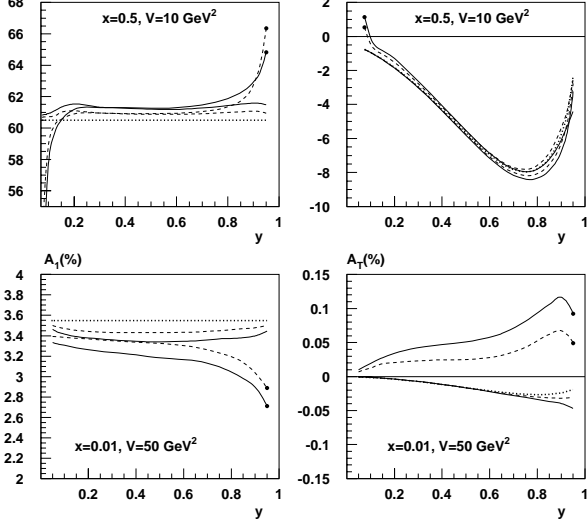


Figure 4: Radiative correction to asymmetries for HERMES (lower plots) and JLab (upper plots) kinematics. Dotted line shows Born asymmetry. Full and dashed line correspond to total and one-loop contributions. Asymmetries with taking into account of elastic contribution are marked by dots in the end

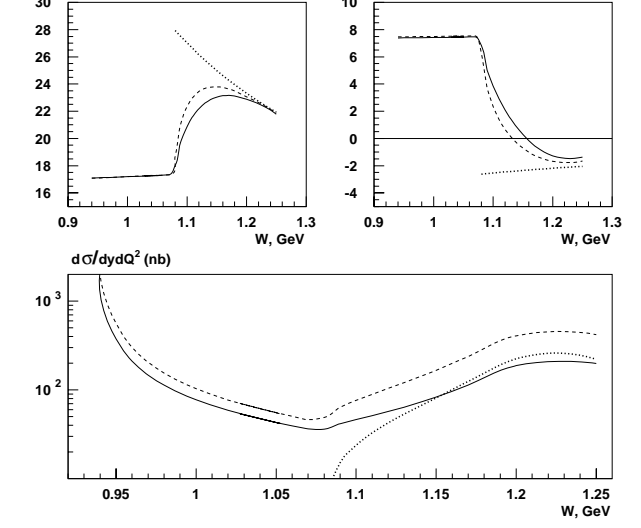


Figure 5: The cross section (lower plot) and polarization asymmetries (both longitudinal and transverse) asymmetries for JLab kinematics ($Q^2=1 \text{ GeV}^2$) near pion threshold. Dotted line shows Born cross section and asymmetry. Full and dashed line correspond to total and one-loop contributions.

also discussion in[9]), and $A_1(x) = x^{0.725}$ suggested in [37]; $g_2 = 0$ (for definition $A_1(x)$ see below).

From these plots we can see that the total radiative correction is basically defined by one-loop correction with some important effect around kinematical borders. The sign and value of the higher order effects is in agreement with leading log estimations and calculations of correction to elastic radiative tail in refs.[38, 39]. Two regions require special consideration, namely, the region of higher y for HERMES and JLab kinematics and the region near pion threshold at JLab.

Let us define the polarization asymmetries as usual

$$A_L = \frac{\sigma_{\parallel}}{\sigma}, \quad A_T = \frac{\sigma_{\perp}}{\sigma}. \quad (44)$$

Also we can define spin asymmetry A_1 which (for chosen model where $g_2 = 0$) is simply related with $A_L = D A_1$, where D is kinematical depolarization factor dependent on the ratio R of longitudinal and transverse photoabsorption cross sections

$$D = \frac{y(2-y)(1+\gamma^2 y/2)}{y^2(1+\gamma^2) + 2(1-y-\gamma^2 y^2/4)(1+R)}, \quad R = \frac{\sigma_L}{\sigma_T} = \frac{M(Q^2 + \nu^2)}{Q^2 \nu} \frac{F_2}{F_1} - 1, .$$

where $\nu = yV/2M$ and $\gamma^2 = Q^2/\nu^2$. For fixed x A_1 is a constant within our model, so it is very convenient for graphical presentation and analysis of different radiative effects. Figure 4 gives asymmetries A_1 and A_T for kinematics of HERMES and JLab up to $y = 0.95$. Influence of higher order and elastic radiative effects can be seen. Figure 5 gives total corrections to cross sections and asymmetries for threshold region of JLab.

7 Conclusion

In this paper we consider model-independent QED radiative correction to the polarized DIS and elastic electron–proton scattering. Together with analytical expression for RC, we give its numerical values for different experimental situations.

Our analytical calculations are based on the electron structure function method which allows to write both the spin-independent and spin-dependent parts of the cross section with accounting of RC to the leptonic part of interaction in the form of well known Drell-Yan representation. The corresponding RC includes explicitly the first order correction as well as the leading-log contribution in all orders of perturbation theory and the main part of the second order next-to-leading-log one. Moreover, any model-dependent RC to the hadronic part of interaction can be included in our analytical result by inserting it as an additive part of the hard cross section under integral sign in master formulae (2) and (13).

To derive RC, we take into account radiation of photons and e^+e^- pairs in collinear kinematics which produces a large logarithm L in the radiation probability (in D -functions) and radiation of one non-collinear photon that enlarges the limits for variation of the hadron structure function arguments. It may be important that these functions are sharp enough. In this case the loss in radiation probability (the loss of L) can be compensated by the increase in the value of the hard cross section.

Note that we extracted the explicit formulae for the first order both with LO and NLO levels. We found analytical agreement between these results for the one-loop correction with the ones known earlier from paper [8], that provides the most important test of total correction.

On the basis of the analytical results, we constructed Fortran code ESFRAD³. Due to several known reasons discussed in Section 6 the results obtained by electron structure method is usually not so convenient for precise numerical analysis. However, we believe that found numerical procedure based on Monte Carlo integration allows us to overcome the obstacles.

Using the developed code we performed numerical analysis for kinematical conditions of current and future polarization experiments. We found two kinematical regions where the higher order radiative correction can be important. These are the traditional region of high y and the region around the pion threshold. We gave detailed analysis of the effects within these regions and presented numerical results within one of the simplest possibility for modeling DIS structure functions. Model dependence of the result is surely an important question requiring a separate investigation for specific application within experimental data analysis.

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³Fortran code ESFRAD is available at <http://www.jlab.org/~aku/RC>

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